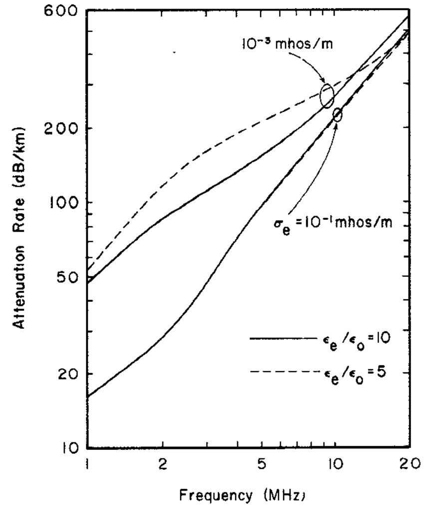

Fig. 8. The effect of tunnel-wall conductivity and permittivity on the attenuation rate of the monofilar mode. (Parameters as in Fig. 6 except for indicated values of $\varepsilon_e/\varepsilon_0$ and σ_e.)

An important related area for further work is the excitation (and reception) of the monofilar and particularly the bifilar mode. Quantitative knowledge is required for a total calculation of system loss and communication range. Also the use of higher frequencies with cables close to the wall merits some attention even though higher attenuation rates can be expected.

References

[1] J. C. Beal, J. Josiak, S. F. Mahmoud, and V. Rawat, "Continuous-access guided communication (CAGC) for ground-transportation systems," *Proc. IEEE* (*Special Issue on Ground Transportation for the Eighties*), vol. 61, pp. 562–568, May 1973.
[2] J. Fontaine, B. DeMoulin, P. deGauque, and R. Gabillard, "Feasibility of radio communication in mine galleries by means of a coaxial cable having a high coupling impedance," in *1973 Proc. Thru-the-Earth Electromagnetics Workshop* (Colorado School of Mines), U.S. Bureau of Mines Contract G-133023, Final Rep. Dec. 31, 1973, pp. 130–139 (available from NTIS, Springfield, VA).
[3] L. Deryck, "Minus et Carriers," *Bulletin Technique* (Institut National des Industries Extractives, Belgium), no. 134, Dec. 1971.
[4] J. R. Wait and D. A. Hill, "Coaxial and bifilar modes on a transmission line in a circular tunnel," *Appl. Phys.*, vol. 4, pp. 307–312, Sept. 1974.
[5] ——, "Propagation along a braided coaxial cable in a circular tunnel," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 401–405, May 1975.
[6] P. Delogne, "The INIEX mine communications system," in *Proc. International Conference Radio: Roads, Tunnels, Mines* (Liege, Belgium), pp. 129–136, April 1974.
[7] V. Rawat and J. C. Beal, "Leaky cables treated as open waveguides," in *Proc. Int. Colloquium Leaky-Feeder Communications* (Guildford, Surrey, England, Apr. 1974).
[8] J. R. Wait and D. A. Hill, "Guided electromagnetic waves along axial conductors in a circular tunnel," *IEEE Trans. Antennas Propagat.* (Succinct Papers), vol. AP-22, pp. 627–630, July 1974.
[9] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1964, p. 378.
[10] A. D. Wheelon, *Tables of Summable Series and Integrals Involving Bessel Functions*. San Francisco: Holden-Day, 1968, p. 37.
[11] J. R. Wait, "Electromagnetic whispering gallery modes in a dielectric rod," *Radio Sci.*, vol. 2, pp. 1005–1017, Sept. 1967.
[12] R. J. Slaughter, "Field leakage and crosstalk, with special reference to radiating cables with perforated tape screens," in *Proc. Int. Colloquium Leaky-Feeder Communications* (Guildford, Surrey, England, April 1974).

Letters

Reflection Coefficient of Unequal Displaced Rectangular Waveguides

RALPH LEVY

***Abstract*—The IEC has suggested that maximum allowable displacements of waveguide flanges should not cause the inherent return loss due to waveguide tolerances to degrade more than 1 dB. Calculations on displaced unequal waveguides at their extreme tolerances show that this leads to a maximum allowable displacement of 0.0175 of the broad (= a) dimension for a waveguide tolerance of $\pm a/500$. The worst return loss under these conditions is approximately −41 dB. However, it is suggested that this maximum allowable displacement is based on a statistically remote worse case condition, and relaxation to a value of 0.021a would be more realistic.**

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The author is with the Microwave Development Laboratories, Inc., Natick, MA 01760.

Introduction

The question of how to specify tolerances on dimensions of locating holes and bolt diameters of rectangular-waveguide flanges, which determine maximum waveguide misalignment, has been under consideration by the International Electro-technical Commission (IEC) Sub-Committee 46B for several years. At their last meeting in Bucharest in 1974 it was proposed that the maximum allowable displacement at a junction of two waveguides shall be such that the degradation of return loss shall be no more than 1 dB relative to the situation where the two waveguides are axially aligned but have different a and b dimensions at extreme tolerances, so that the worst possible reflection coefficient results. Thus, if the tolerances on the a and b dimensions are $\pm\Delta a$ and $\pm\Delta b$, for the worst possible reflection coefficient at the axially aligned junction, one waveguide will have dimensions $a + \Delta a$, $b - \Delta b$, and the other $a - \Delta a$, $b + \Delta b$.

This choice of constraint factor for misalignment tolerance seems more logical than previous factors which involved an arbitrary choice of worst allowable return loss for displaced identical waveguides. It recognizes that in practice the waveguides are not identical and do indeed possess finite return loss even when perfectly aligned, so that there is no point in choosing a figure for displacement return loss which is much better than the axially aligned return loss. The "1-dB-worse" condition then becomes a more practical and less arbitrary criterion for determining the constraints.

It is necessary therefore to develop formulas for the return loss of displaced unequal waveguides. Initially, the reflection coefficient in the axially aligned case must be determined. The relative impedance of almost equal waveguides is given by [1], [2]

$$Z_0 \propto \lambda_g b \tag{1}$$

where λ_g is the guide wavelength and b is the waveguide narrow dimension. From (1) an elementary calculation gives the reflection coefficient at the junction of a pair of waveguides at their extreme tolerances $a \pm \Delta a$ and $b \pm \Delta b$ as

$$\begin{aligned} \rho_z &= \left(\frac{\lambda g}{2a}\right)^2 \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \\ &= \left(\frac{\lambda g}{2a}\right)^2 \frac{\Delta a}{a} + \frac{\Delta b}{b} \text{ (for worst case)} \end{aligned} \tag{2}$$

where ρ_z is real at the junction, and examination of published formulas for the reactive terms, e.g., [1], shows that for any reasonable small waveguide tolerances these are quite negligible, being proportional to the square of $\Delta a/a$ and $\Delta b/b$. For example, the contribution to the shunt susceptance due to the displacement $\pm\Delta a$ is given, by simplication of Marcuvitz [1, (2c), p. 296] and substituting $\beta = 2\Delta a/a$, as

$$\rho_a \simeq \frac{1}{2}\frac{B}{Y_0} \simeq \frac{\lambda g}{a}\left(\frac{\Delta a}{a}\right)^2 \ln\frac{a}{\Delta a}. \tag{3}$$

It should be mentioned that in this case it is valid to calculate the effect of tolerances and displacement independently for the a and b dimensions, and to add them vectorially [2].

The 1-dB-Worse Condition

The reflection coefficient ρ_d due to a displacement from the axially aligned condition is in phase quadrature with the resistive component (2), which remains unchanged by such displacement. The combined reflection coefficient modulus, ρ_t, is

$$\rho_t = \sqrt{\rho_z^2 + \rho_d^2} \tag{4}$$

giving a reflection (return) loss of

$$D_t = 20 \log_{10} \rho_t = 10 \log (\rho_z^2 + \rho_d^2). \tag{5}$$

The return loss for axially aligned waveguide is

$$D_z = 20 \log \rho_z. \tag{6}$$

Hence the condition for a 1-dB degradation in D_z due to a waveguide displacement (remembering that D_z is negative) is

$$\rho_z^2 + \rho_d^2 = \text{antilog}\,\frac{D_z + 1}{10}. \tag{7}$$

Substituting for ρ_z from (6) gives

$$\rho_d^2 = \text{antilog}\,\frac{D_z + 1}{10} - \text{antilog}\,\frac{D_z}{10}$$

so that the displacement return loss is

$$D_d = 10 \log \rho_d^2 = D_z - 5.86825. \tag{8a}$$

It is proposed that for simplicity this should be taken as

$$D_d = D_z - 6. \tag{8b}$$

Thus, if the axially aligned junction gives a -40-dB return loss, this degrades to -39 dB if a displacement takes place equivalent to a reactive return loss of -46 dB.

Reflection Coefficient of Misaligned Unequal Waveguides

The reflection coefficient of displaced *equal* waveguides has been evaluated by a number of workers, including Kienlin and Kürzl [3] and Lucas [4]. It is proposed to use the simple but quite accurate formulas of the former [3], namely

$$\rho_a = \frac{\pi^2 \lambda g}{2a}\left(\frac{\Delta a'}{a}\right)^2 \tag{9}$$

$$\rho_b = \frac{\pi^2 b}{1.25\lambda g}\left(\frac{\Delta b'}{b}\right)^2. \tag{10}$$

The experimental data presented in [3] show these formulas to give return losses accurate to within 0.5 dB for $\Delta b'/b < 0.3$ and $\Delta a'/a < 0.15$, corresponding to extreme return losses of approximately -15 dB. The accuracy appears to be much better than 0.5 dB for the rather small displacements considered here. In addition to these contributions from displacements $\Delta a'$ and $\Delta b'$ there is a contribution from a possible angular twist, but this may be shown to be too small to be of any consequence [5].

Displacement of *equal* waveguides in either a or b dimensions gives rise to a pair of equal shunt susceptances in parallel, one due to each step at opposite sides of the waveguide. For the very small discontinuities considered here, it is valid to take the reflection coefficient of each step as being equal to half the value of the total normalized susceptance. Therefore each side of the waveguide is responsible for half of the reflection coefficient due to the displacement.

In the case of displaced *unequal* waveguides it is necessary to modify (9) and (10) to take the inequalities into account. Consider the case where the waveguides have broad dimensions $a + \Delta a$ and $a - \Delta a$, and are axially aligned. Upon displacement by an amount $\Delta a'$, the steps at the two sides become $\Delta a' + \Delta a$ and $\Delta a' - \Delta a$. Hence from the previous argument and (9) it follows that the reflection coefficient due to the displacement is

$$\begin{aligned} \rho_a &= \frac{\pi^2 \lambda_g}{2a} \frac{\left(\frac{\Delta a' + \Delta a}{a}\right)^2 + \left(\frac{\Delta a' - \Delta a}{a}\right)^2}{2} \\ &= \frac{\pi^2 \lambda_g}{2a}\left[\left(\frac{\Delta a'}{a}\right)^2 + \left(\frac{\Delta a}{a}\right)^2\right]. \end{aligned} \tag{11}$$

Similarly, for a displacement $\Delta b'$ in the b dimension

$$\rho_b = \frac{\pi^2 b}{1.25\lambda_g}\left[\left(\frac{\Delta b'}{b}\right)^2 + \left(\frac{\Delta b}{b}\right)^2\right]. \tag{12}$$

Since the reflection coefficients (11) and (12) are actually of opposite sign, the worst case reflection coefficient occurs for displacement in one or the other direction, not both simultaneously. At the low end of the waveguide operating frequency band, displacement in the a dimension causes the greater reflection coefficient and vice versa.

It is now possible to derive simple one-line formulas for the maximum allowable a and b dimensional displacements, each resulting in a 1-dB degradation from the axially aligned case. For a displacement in the waveguide broad dimension the ρ_a in (8) is equal to the ρ_a in (11), i.e.,

$$\rho_a = \rho_a = \text{antilog}\,\frac{D_z - 5.86825}{20} = 0.50885\ 10^{D_z/20} \tag{13}$$

which using (6) becomes related to the reflection coefficient of the axially aligned waveguide ρ_z by

$$\rho_a = 0.50885\rho_z. \tag{14}$$

Note that for the sake of precision (8a) has been used here rather than the simple "rule-of-thumb" approximation (8b).

The "worse case" ρ_z is given from (2) as

$$\rho_z = \left(\frac{\lambda_g}{2a}\right)^2 \cdot \frac{\Delta a}{a} + \frac{\Delta b}{b}. \tag{15}$$

Substituting for ρ_a in (11) from (14) and (15) and rearranging gives the desired formula in the form

$$\frac{\Delta a'}{a} = \sqrt{\frac{0.50885}{\pi^2}\,\frac{2a}{\lambda_g}\left[\left(\frac{\lambda_g}{2a}\right)^2 \frac{\Delta a}{a} + \frac{\Delta b}{b}\right] - \left(\frac{\Delta a}{a}\right)^2}. \tag{16}$$

Similarly, the formula for the maximum allowable displacement in the waveguide narrow dimension is

$$\frac{\Delta b'}{b} = \sqrt{\frac{0.50885}{\pi^2}\cdot\frac{1.25\lambda_g}{b}\left[\left(\frac{\lambda_g}{2a}\right)^2 \frac{\Delta a}{a} + \frac{\Delta b}{b}\right] - \left(\frac{\Delta b}{b}\right)^2}. \tag{17}$$

Here the tolerances on the waveguide dimensions are $\pm\Delta a$, $\pm\Delta b$ (usually $\Delta a = \Delta b$), and λ_g is the guide wavelength. It is seen that these maximum fractional displacements are a function only of guide wavelength, aspect ratio a/b, and fractional waveguide dimensional tolerances.

The IEC has fixed maximum waveguide tolerances from waveguide sizes R40 through R220 at the value $\Delta a = \Delta b = \pm a/500$. Substituting this value in (16) and (17) gives the values of $\Delta a'/a$ and $\Delta b'/b$ for various values of f/f_c in Table I, for aspect ratio $a/b = 2.25$, and the aforementioned waveguide tolerance. The final two columns give the permissible fractional displacement in either the a direction or the b direction.

Thus for R100 where $a = 0.900$ in, $b = 0.400$ in, the maximum permissible displacement at the low end of the band where $f/f_c = 1.25$ is determined by the a displacement, and is 0.016 in. At the high-frequency end it is determined by the b displacement, and is 0.012 in. Hence the most restrictive tolerance is determined by the b displacement at the high-frequency end of the band, but this is where the misaligned return loss has its best value. Hence it is arguable that the worst tolerance here should be 0.016 in not 0.012 in, since the return loss would not fall below -40.88 dB anywhere in the band for the larger displacement. Using this criterion, the appropriate values are tabulated in Table II for IEC waveguide sizes from R40 through R220. The final column gives the poorest permissible return loss resulting from the maximum displacement, which occurs at the low-frequency end of the operating range.

TABLE I
RETURN LOSS AND MAXIMUM DISPLACEMENT OF MISALIGNED UNEQUAL WAVEGUIDES

f/f$_c$	RETURN LOSS (dB)		PERMISSIBLE FRACTIONAL DISPLACEMENT	
	Waveguides aligned	Waveguides misaligned	Δa'/a	Δb'/b
1.25	-41.88	-40.88	.0175	.0556
1.4	-43.63	-42.63	.0181	.0439
1.5	-44.29	-43.29	.0186	.0395
1.7	-45.10	-44.10	.0200	.0339
1.9	-45.57	-44.57	.0209	.0304

Note: Aspect ratio $a/b = 2.25$; waveguide tolerance $\Delta a = \Delta b = \pm a/500$.

TABLE II
PERMISSIBLE FLANGE DISPLACEMENTS ACCORDING TO THE IEC

IEC waveguide designation	Aspect ratio a/b	Maximum Permissible flange displacement			Poorest permissible return loss
		Δa'/a	Δa' (in.)	Δa' (mm.)	(dB)
R40	2.000	0.0170	0.039	0.989	-41.4
R48	2.1468	0.0173	0.032	0.823	-41.1
R58	2.000	0.0170	0.027	0.687	-41.4
R70	2.2058	0.0174	0.024	0.606	-41.0
R84	2.2575	0.0175	0.020	0.499	-40.9
R100	2.250	0.0175	0.016	0.400	-40.9
R120	2.000	0.0170	0.013	0.324	-41.4
R140	2.000	0.0170	0.011	0.269	-41.4
R180	2.000	0.0170	0.0087	0.220	-41.4
R220	2.4706	0.0180	0.0076	0.192	-40.4

Note: "1-dB-worse" criterion for waveguide tolerance $\Delta a = \Delta b = \pm a/500$.

Since the value of $\Delta a'/a$ varies by less than ± 3 percent from a mean value for all waveguide sizes, the entire table may be summarized by a rule giving the maximum allowable flange displacement as

$$\Delta a' = 0.0175a \text{ (worst case)} \tag{18}$$

for all aspect ratios, leading to a worst return loss of approximately -41 dB at the low end of the recommended frequency range. This is in accordance with the IEC waveguide tolerance of $\pm a/500$ and its recommended 1-dB degradation criterion.

The values of displacement given in Table II or by (18) are for a worst case condition. It should be realized, however, that if it were possible to use a statistical approach a greater design displacement would be permissible. This becomes apparent when it is realized that the displacement is determined by the following tolerances:

1) a circular positional tolerance on the flange holes;
2) a tolerance on the flange-hole diameters;
3) a tolerance on the waveguide inside dimensions;
4) the difference between the maximum hole diameter and the minimum locating bolt diameter.

The probability of two waveguides being aligned with only a or only b displacements is therefore quite small. An additional consideration is that one is usually concerned with the overall return loss of several flanges in a waveguide run, and the probability that all flanges will be misaligned similarly is even more remote.

The original criterion of basing the initial aligned return loss on the case where the waveguides have extreme tolerances $a + \Delta a, b - \Delta b$ and $a - \Delta a, b + \Delta b$ is also a statistically remote condition. Usually, the return loss will be much better. In fact, in the case where the two waveguides have a negligible impedance discontinuity, the return losses for the displacements shown in Table II are 6 dB better. Alternatively, one can state that in this case the deviation required to give the return loss values allowed in Table II is approximately 0.025a, a figure significantly larger than that of (18), i.e.,

$$\Delta a' = 0.025a \text{ (matched-waveguide case).} \qquad (19)$$

In practice it is found that while it may be quite difficult to dimension the flanges to ensure that (18) is not exceeded, an allowable displacement approximately midway between the 0.0175a of (18) and the 0.025a of (19) is more readily feasible. Such a compromise value of 0.21a gives a worst return loss of approximately −40 dB. It makes reasonable allowance for the statistical considerations, enables reasonable tolerances to be assigned in most instances, and is essentially in accordance with the basic logic behind the IEC 1-dB degradation criterion.

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REFERENCES

[1] *Waveguide Handbook*, N. Marcuvitz, Ed. New York: McGraw-Hill, 1951, p. 296.
[2] H. J. Riblet, "A general design procedure for quarter-wavelength inhomogeneous transformers having approximately equal-ripple performance," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp. 622–629, September 1965.
[3] U. Von Kienlin and A. Kürzl, "Reflexionen an Hollheiter-Flanschverbindungen," *Nachriehtentech*, vol. 11, pp. 561–564, 1958.
[4] I. Lucas, "Reflexionsfaktoren an Versetzungen in Rechteckhohlleitern," *Arch. Elek. Uebertragungen*, vol. 20, no. 12, pp. 683–690, 1966.
[5] H. A. Wheeler and H. Schwiebert, "Step-twist waveguide components," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-3, pp. 44–52, October 1955.

A 60-W CW Solid-State Oscillator at C Band

R. N. WALLACE, M. G. ADLERSTEIN, AND S. R. STEELE

Abstract—**A 60-W CW solid-state oscillator has been developed for operation in C band. The oscillator combines the power of six high-efficiency GaAs multimesa Read diodes. Single-diode oscillators have given power outputs as high as 13.3-W CW at 5 GHz.**

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The authors are with the Raytheon Research Division, Waltham, MA 02154.

A solid-state oscillator delivering 60-W nominal CW power in C band has been constructed and tested. The oscillator circuit combines the outputs from six high-efficiency multimesa GaAs Read IMPATT diodes. Such diodes, incorporating chip-level power combining, allow one to reach the 60-W level with relatively few discrete devices. Compared to an oscillator combining a large number of low-power diodes, the source described here operates with simpler bias circuitry and is easier to tune for optimum performance.

Fabrication of the high-power diodes from low-high-low Read profile epitaxial GaAs wafers grown in our laboratory has been described in some detail elsewhere [1]. Individual diodes consist of four separate mesas in a 2 × 2 array, mounted on an integral plated-gold heat sink. This arrangement provides a substantial improvement in thermal resistance over that obtained with a single mesa of equivalent area [2], and we regularly measure values of 4.5–5.0°C/W in C-band diodes. With a room-temperature heat sink, a diode having 22–23-percent efficiency can thus produce 10-W CW output with a junction temperature of ~200°C. The plated heat-sink technology is suited for large-volume, low-cost diode production, and offers economic advantages over the IIa diamond heat sinks often used in obtaining high-power operation [3].

The yield of diodes producing 10-W CW or more is not yet large, but our results indicate that the devices will be manufacturable. In a recent series of thirteen epitaxial wafers selected for processing, eleven produced diodes which, when operated with 40°C (nominal) heat-sink temperature, reached or exceeded 10-W CW output. Efficiencies were typically 22–25 percent, and frequencies of optimum performance ranged from 4.7 to 6.5 GHz. Two wafers, grown in different epitaxial reactors, produced best results of 13.3-W CW output with 24-percent efficiency in the 4.8–4.9-GHz range. A 15.3-W CW result with a cooled heat sink was reported previously [1]. These are among the highest power outputs reported for C-band IMPATT diodes.

The six-diode oscillator circuit used in the present work is similar to the type described by Harp and Stover [4]. The circuit concept has been analyzed in considerable detail and tested experimentally by Kurokawa [5], [6]. A few practical operating considerations will be noted here.

Fig. 1 is a schematic representation of the oscillator circuit. The basic resonator is a cylindrical TM_{010}-mode cavity. The cavity frequency is adjusted with a dielectric rod tuner, and coupling to the external load is controlled by varying the penetration of the coaxial output probe. The six diodes are coupled to the main resonator through coaxial lines ($Z_0 = 50\ \Omega$) passing along the cavity side wall. Coupling between the diodes and the cavity is adjusted by moving the diode mounting plugs axially, and by changing the dimensions of the individual slug transformers. Bias is supplied to the diodes along the coaxial center conductors, which pass through absorbing terminations at the top of the cavity. The circuit is water cooled during operation.

The operation and tuning of the circuit can be conveniently described in terms of the impedance Z_m measured on the diode coaxial lines at the midplane of the cavity. For frequencies near resonance, this impedance is essentially that of a parallel RLC circuit (the loaded cavity) in series with a resistor (the terminated bias line). At resonance, Z_m is resistive and may range from $\sim 1.5Z_0$ to $\sim 20Z_0$ depending on the output coupling adjustment.

Large-signal terminal impedances of the individual diodes are approximately $-0.8 + j6\ \Omega$ near 5 GHz. The slug and coaxial line must thus be designed to transform Z_m to the much lower impedance required by the diode. The range of adjustments